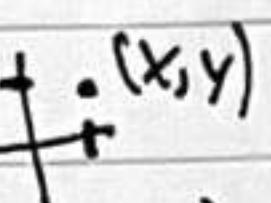
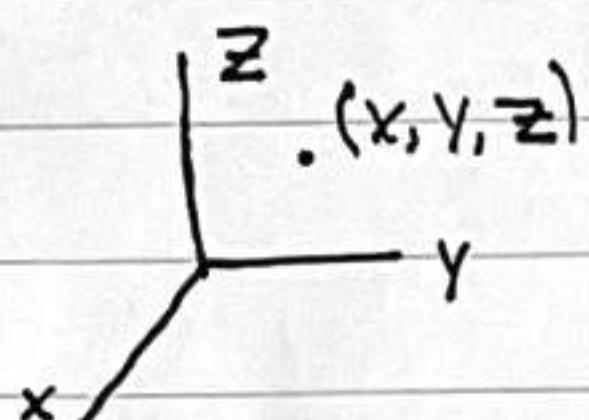


2/25

12.1 - Coordinates in 3-space (\mathbb{R}^3)

\mathbb{R}^2 2-space: 
 $\mathbb{R}^2 = \{(x, y) : x, y \in \mathbb{R}\}$

\mathbb{R}^3 3-space: 
A point (x, y, z) is located in the 3D space.

$$\mathbb{R}^3 = \{(x, y, z) : x, y, z \in \mathbb{R}\}$$

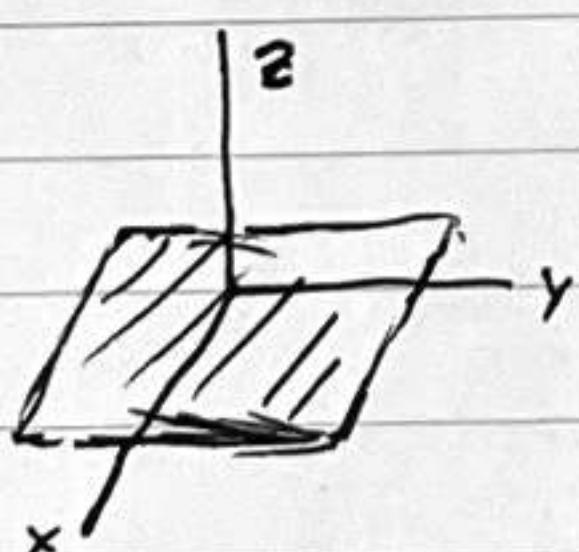
The coordinates of a point (x, y, z) are $x, y, z \dots$

I. Coordinate planes

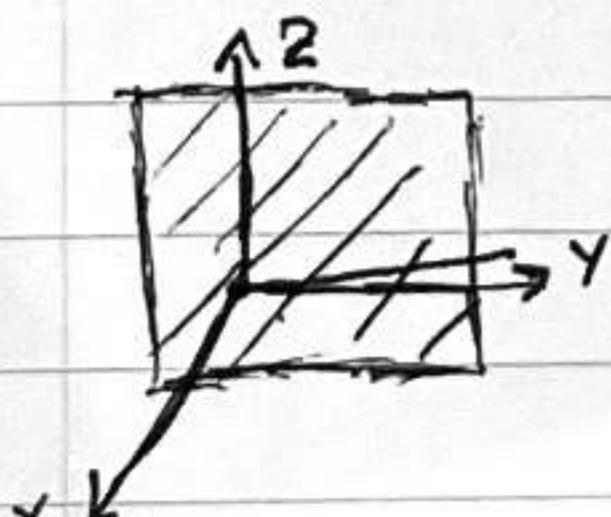
A coordinate plane in \mathbb{R}^3 is a set of points, all of which have a (predetermined) coordinate set to 0.

ex/

The XY plane in \mathbb{R}^3 is the $z=0$ plane, i.e. $\{(x, y, z) \in \mathbb{R}^3 : z=0\}$



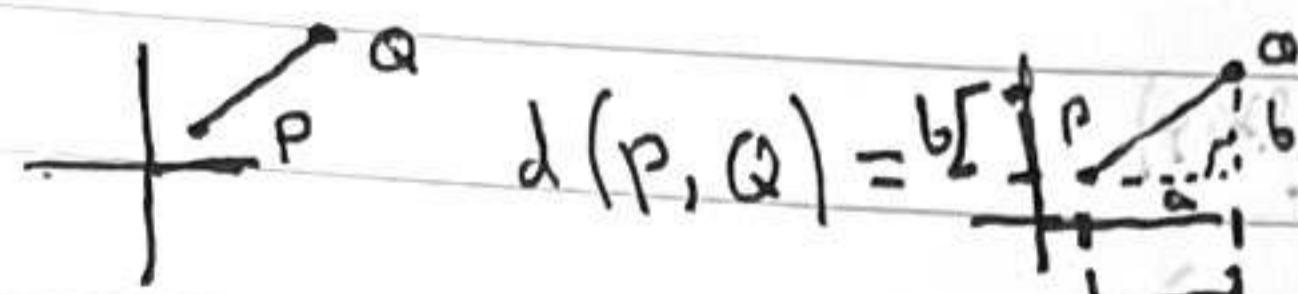
The YZ plane in \mathbb{R}^3 is the $x=0$ plane, i.e. $\{(x, y, z) \in \mathbb{R}^3 : x=0\}$



Distances in 3-space

2 space

$$\mathbb{R}^2:$$

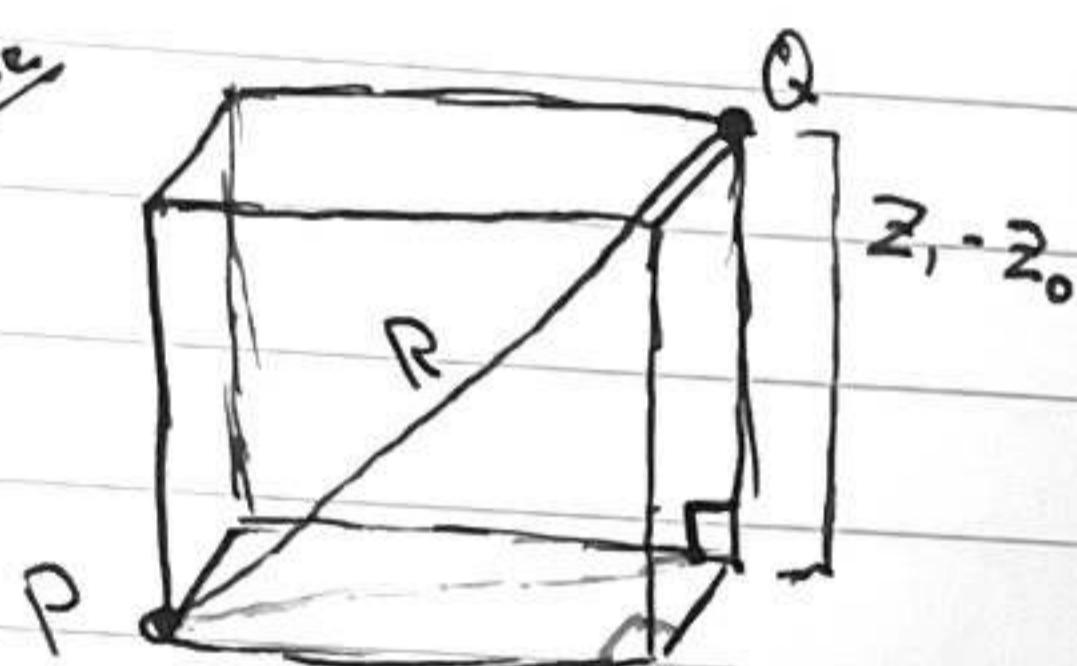


$$d(P, Q) = \sqrt{a^2 + b^2}$$

$$d(P, Q) = \sqrt{a^2 + b^2} = \sqrt{(x_0 - x_p)^2 + (y_0 - y_p)^2}$$

$$d(P, Q) = \sqrt{(x_0 - x_p)^2 + (y_0 - y_p)^2}$$

3 space



$$z_1 - z_0$$

$$\sqrt{x_1 - x_0} \times \sqrt{y_1 - y_0} = \sqrt{R^2 - (z_1 - z_0)^2}$$

$$R = \sqrt{(z_1 - z_0)^2 - (\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2})^2}$$

$$R = \sqrt{(z_1 - z_0)^2 + (x_1 - x_0)^2 + (y_1 - y_0)^2}$$

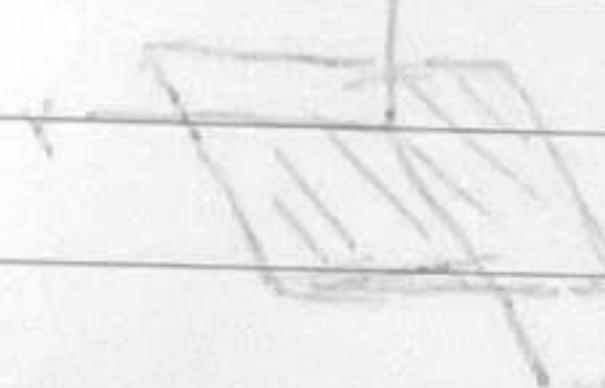
Thm: Let $P = (x_0, y_0, z_0)$ and $Q = (x_1, y_1, z_1)$ be points in \mathbb{R}^3

The distance between them is

$$d(P, Q) = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$$

$\Rightarrow \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2} = \sqrt{x_1^2 + y_1^2 + z_1^2 - (x_0^2 + y_0^2 + z_0^2)}$

5



5

